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- One-time cost on the large set and linear recurrent cost on the smaller set ۲
- Proposed optimizations to reduce computation time and communication ۲
- Reduce intersection time by one/two OOM compared to SOTA on slow/fast networks ۲

Recurrent time (s) for $ X \approx 2^{20}$, $ Y = 4$							
Protocol	10 Gbps	100 Mbps					
This work	0.03	0.24					
CLR [1]	2.21	2.81					
KKRT [2]	1.67	6.46					
PSSZ [3]	0.88	2.94					

[1] Hao Chen, Kim Laine, and Peter Rindal. "Fast private set intersection from homomorphic encryption." CCS, 2017.

[2] Vladimir Kolesnikov, Ranjit Kumaresan, Mike Rosulek, and Ni Trieu. "Efficient batched oblivious PRF with applications to private set intersection." CCS, 2016.

[3] Benny Pinkas, Thomas Schneider, Gil Segev, and Michael Zohner. "Phasing: Private set intersection using permutation-based hashing." USENIX Security, 2015.

Proposed PSI Protocol

Protocol 1 Basic Protocol

Input: S inputs set X and \mathcal{R} inputs set Y. Both sets consist of bit strings of length σ . Sizes |X|, |Y|, and σ are public. **Output**: S outputs \perp ; \mathcal{R} outputs $X \cap Y$.

- (1) Setup: Parties agree on a Fully Homomorphic Encryption scheme and encryption parameters (n, q, t).
- 2) Set encryption: S encrypts each element $x_i \in X$ under its own key, and sends the |X| ciphertexts $\overline{X} = (\overline{x_1}, ..., \overline{x_{|X|}})$ to \mathcal{R} .
- 3) **Computing intersection**: For each $y_j \in Y$, \mathcal{R} :
 - a) samples a random plaintext $r_j \in \mathbb{Z}_t$ from a uniform distribution;
 - b) homomorphically computes

$$\overline{d_j} = r_j + \prod_{i=1}^{|X|} (\overline{x_i} - y_j)$$

- c) encrypts $r_j \mapsto \hat{r_j}$ under its own key;
- d) sends $\overline{d_j}$ and $\widehat{r_j}$ to \mathcal{S} .
- 4) **Decryption**: For each $(\overline{d_j}, \hat{r_j}), S$:
 - a) decrypts $\overline{d_j} \mapsto d_j$;
 - b) samples a non-zero random plaintext value $\rho_i \in \mathbb{Z}_t \setminus \{0\}$ from a uniform distribution;
 - c) homomorphically computes

$$\widehat{e_j} = (\widehat{r_j} - d_j) \cdot \rho_j$$

- d) sends $\widehat{e_j}$ to \mathcal{R} .
- 5) **Result**: \mathcal{R} decrypts $\widehat{e_j} \mapsto e_j$ and outputs

$$X \cap Y = \bigcup_{j=1}^{|Y|} \{ y_j : e_j = 0 \}$$

Receiver

Sender

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Receiver

Sender

encrypts set with Sender's key

sends encrypted set

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- 5) **Result**: \mathcal{R} decrypts $\hat{e_j} \mapsto e_j$ and outputs

$$X \cap Y = \bigcup_{j=1}^{|Y|} \{ y_j : e_j = 0 \}$$

Receiver

Sender

encrypts set with Sender's key



computes intersection

adds random value

encrypts random value with Receiver's key

> sends encrypted result and encrypted random value

Protocol 1 Basic Protocol

Input: S inputs set X and \mathcal{R} inputs set Y. Both sets consist of bit strings of length σ . Sizes |X|, |Y|, and σ are public. **Output**: S outputs \bot ; \mathcal{R} outputs $X \cap Y$.

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$$\overline{d_j} = r_j + \prod_{i=1}^{|X|} (\overline{x_i} - y_j)$$

c) encrypts
$$r_j \mapsto \hat{r_j}$$
 under its own key

d) sends
$$\overline{d_j}$$
 and $\hat{r_j}$ to \mathcal{S} .

4) Decryption: For each (d_j, r̂_j), S:
a) decrypts d_j → d_j;
b) samples a non-zero random plaintext value ρ_j ∈ Z_t \{0} from a uniform distribution;
c) homomorphically computes
ê_j = (r̂_j - d_j) · ρ_j
d) sends ê_j to R.
5) Result: R decrypts ê_j → e_j and outputs
X ∩ Y = ∪^{|Y|}_{i=1} {y_i : e_i = 0}

Receiver

Sender

encrypts set with Sender's key

sends encrypted set

computes intersection

adds random value

encrypts random value

with Receiver's key

sends encrypted result and encrypted random value

decrypts result

subtracts random value

```
multiply by another random value
```

sends final result

Protocol 1 Basic Protocol

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d) sends
$$\widehat{e_j}$$
 to \mathcal{R} .

5) **Result**: \mathcal{R} decrypts $\hat{e_j} \mapsto e_j$ and outputs $X \cap Y = \bigcup_{i=1}^{|Y|} \{y_j : e_j = 0\}$

Receiver

Sender

encrypts set with Sender's key

sends encrypted set

computes intersection adds random value encrypts random value with Receiver's key sends encrypted result and encrypted random value decrypts result subtracts random value multiply by another random value

decrypts final result checks if value is zero or not

P	rotoc	col 1 Basic Protocol		
Inp	ut:	\mathcal{S} inputs set X and \mathcal{R} inputs set Y. Both sets consist		
of l	oit st	rings of length σ . Sizes $ X $, $ Y $, and σ are public.	Receiver	Sender
Ou	1)	Solution \perp , \mathcal{R} outputs $A + H$.		
	2)	Setup: Parties agree on a Fully Homomorphic Encryption scheme and encryption parameters (n, q, t) . Set encryption: S encrypts each element $x_i \in X$ under its own key, and sends the $ X $ ciphertexts $\overline{X} = (\overline{x_1},, \overline{x_{ X }})$ to \mathcal{R} .	One-time	encrypts set with Sender's key
ſ	3)	Computing intersection : For each $y_j \in Y$, \mathcal{R} :	computes intersection	
		 a) samples a random plaintext r_j ∈ Z_t from a uniform distribution; b) homomorphically computes 	adds random value	
			encrypts random value	
		$\overline{d_j} = r_j + \prod_{i=1}^{ T } (\overline{x_i} - y_j)$	with Receiver's key	
		c) encrypts $r_i \mapsto \hat{r_i}$ under its own key:	sends encrypted re	esult and
		d) sends $\overline{d_j}$ and $\widehat{r_j}$ to \mathcal{S} .	encrypted randot	n value
	4)	Decryption : For each $(\overline{d_j}, \widehat{r_j}), S$:	Recurrent	
		a) decrypts $\overline{d_j} \mapsto d_j$;		decrypts result
		b) samples a non-zero random plaintext value		subtracts random value
		c) homomorphically computes		
		$\hat{\boldsymbol{x}} = (\hat{\boldsymbol{x}} - d)$		multiply by another random value
		$e_j = (r_j - a_j) \cdot \rho_j$ d) sends $\hat{e_j}$ to \mathcal{R}	sends final re	sult
	5)	Result : \mathcal{R} decrypts $\widehat{e_j} \mapsto e_j$ and outputs	decrypts final result	
		$X \cap Y = \bigcup_{j=1}^{ Y } \{ y_j : e_j = 0 \}$	checks if value is zero or not	

Protocol 1 Basic Protocol		Assuming a Sender's set with one million e
Input: S inputs set X and \mathcal{R} inputs set Y. Both sets consist of bit strings of length σ . Sizes $ Y Y $ and σ are public	D .	0 1
Output: S outputs \perp ; \mathcal{R} outputs $X \cap Y$.	Receiver	Sender
 Setup: Parties agree on a Fully Homomorphic Encryption scheme and encryption parameters (n, q, t). Set encryption: S encrypts each element x_i ∈ X under its own key, and sends the X ciphertexts X = (x_i → x_i) to R 	One-time	encrypts set with Sender's key <u>oted set</u> ~ 7 Terabytes
 3) Computing intersection: For each y_j ∈ Y, R: a) samples a random plaintext r_j ∈ Z_t from a uniform distribution; b) homomorphically computes 	computes intersection Linear so adds random value	earch (≈ 1 million multiplications)
c) encrypts $r_j \mapsto \hat{r_j}$ under its own key;	encrypts random value with Receiver's key sends encrypted	result and
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$\widehat{e_j} = (\widehat{r_j} - d_j) \cdot \rho_j$ d) sends $\widehat{e_j}$ to \mathcal{R} . 5) Result : \mathcal{R} decrypts $\widehat{e_j} \mapsto e_j$ and outputs	decrypts final result	result
$X \cap Y = \bigcup_{j=1}^{ I } \{ y_j : e_j = 0 \}$	checks if value is zero or not	

tries

Solving The Linear Search Problem

Sender hashes set X before sending it to Receiver.

Solving The Linear Search Problem

Sender hashes set X before sending it to Receiver.

Constraint: All bins must have the same number of items.

Simple hashing



Hash functions: **h**

Maximum load $O\left(\frac{\log n}{\log \log n}\right)$

Number of entries $(n \log n)$



Multiplicative depth $O\left(\log \frac{\log n}{\log \log n}\right)$

Solving The Linear Search Problem

Sender hashes set X before sending it to Receiver.

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Simple hashing







Number of entries $O\left(\frac{n\log n}{\log\log n}\right)$

Multiplicative depth $O\left(\log \frac{\log n}{\log \log n}\right)$





Hash functions: $h_1, \cdots, h_{|H|}$

Maximum load 1

Number of entries $\leq 2n(1 + \epsilon)$

Multiplicative depth O(log|H|)

Protocol 1 Basic Protocol		Assuming a Sender's set with one million e
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Output: S outputs \perp ; \mathcal{R} outputs $X \cap Y$.	Receiver	Sender
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	under its own key, and sends the $ X $ ciphertexts $X = (\overline{x_1},, \overline{x_{ X }})$ to \mathcal{R} .	sends e	$encrypted set \approx 100 \text{ Gigabytes}$
3	 Computing intersection: For each y_j ∈ Y, R: a) samples a random plaintext r_j ∈ Z_t from a uniform distribution; b) homomorphically computes 	computes intersection Cor adds random value	nstant-time search (3 multiplications)
	$\overline{d_j} = r_j + \prod_{i=1}^{ X } (\overline{x_i} - y_j)$	encrypts random value with Receiver's key	
	c) encrypts $r_j \mapsto \hat{r_j}$ under its own key;	sends encr	rypted result and
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5	$\widehat{e_j} = (\widehat{r_j} - d_j) \cdot \rho_j$ d) sends $\widehat{e_j}$ to \mathcal{R} .) Result : \mathcal{R} decrypts $\widehat{e_j} \mapsto e_j$ and outputs	decrypts final result	s final result
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tries

Pack many entries into one ciphertext.

Pack many entries into one ciphertext.

FHE encrypts plaintext polynomials into ciphertext polynomials.

Message space: $M = \mathbb{Z}_t$ Plaintext space: $P = \mathbb{Z}_t[x]/(x^n + 1)$ Ciphertext space: $C = \mathbb{Z}_q[x]/(x^n + 1)$

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Integer $p = \varepsilon(m) : M \mapsto P$ $c = E(p) : P \mapsto C^2$

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Integer	Batching ^[4]
$p = \varepsilon(m) : M \mapsto P$	$p = \varepsilon(m_1, m_2, \cdots, m_n) : M^n \mapsto P$
$c = E(p) : P \mapsto C^2$	$c = E(p) : P \mapsto C^2$

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IntegerBatching [4]Batching + Packing
$$p = \varepsilon(m) : M \mapsto P$$
 $p = \varepsilon(m_1, m_2, \cdots, m_n) : M^n \mapsto P$ $p = \varepsilon(m_1, m_2, \cdots, m_{kn}) : M^{kn} \mapsto P$ $c = E(p) : P \mapsto C^2$ $c = E(p) : P \mapsto C^2$ $c = E(p) : P \mapsto C^2$

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We encode Cuckoo hash table T with proposed "Batching + Packing" method.

≈ 12.5 Megabytes(Unencrypted is 4 MB)

[4] Nigel P. Smart and Frederik Vercauteren. "Fully homomorphic SIMD operations." Designs, codes and cryptography, 2014.



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	under its own key, and sends the $ X $ ciphertexts $X = (\overline{x_1},, \overline{x_{ X }})$ to \mathcal{R} .	sends e	$encrypted set \approx 100 \text{ Gigabytes}$
3	 Computing intersection: For each y_j ∈ Y, R: a) samples a random plaintext r_j ∈ Z_t from a uniform distribution; b) homomorphically computes 	computes intersection Cor adds random value	nstant-time search (3 multiplications)
	$\overline{d_j} = r_j + \prod_{i=1}^{ X } (\overline{x_i} - y_j)$	encrypts random value with Receiver's key	
	c) encrypts $r_j \mapsto \hat{r_j}$ under its own key;	sends encr	rypted result and
4	d) sends $\overline{d_j}$ and $\widehat{r_j}$ to S .) Decryption : For each $(\overline{d_j}, \widehat{r_j})$, S : a) decrypts $\overline{d_j} \mapsto d_j$; b) samples a non-zero random plaintext value $\rho_j \in \mathbb{Z}_t \setminus \{0\}$ from a uniform distribution; c) homomorphically computes	Recurrent encrypted	d random value decrypts result subtracts random value multiply by another random value
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Proto	col 1 Basic Protocol		Assuming a Sender's set with one million e
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4)	 Decryption: For each (d_j, r̂_j), S: a) decrypts d_j → d_j; b) samples a non-zero random plaintext value ρ_j ∈ Z_t \{0} from a uniform distribution; c) homomorphically computes 	Recurrent	decrypts result subtracts random value multiply by another random value
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	$X \cap Y = \bigcup_{j=1}^{ Y } \{ y_j : e_j = 0 \}$	checks if value is zero or not	

tries

Results

			$ \mathbf{V} = \mathscr{Q} \cdot 2^{20}$					Y	
			$ \Lambda = \mathcal{I} \cdot \mathcal{I}^{-1}$				4	16	64
		Fast Setup	0.09			Fast Setup	0.02	0.11	0.43
		Fast Intersection	0.30			Fast Intersection	0.03	0.10	0.37
	Computation				Computation				
		Fast Setup	12.63			Fast Setup	1.85	7.39	29.55
		Fast Intersection	25.25			Fast Intersection	0.66	2.65	10.59
	Communication				Communication				
One-time				Recurrent					
		Fast Setup	0.10			Fast Setup	0.02	0.12	0.45
	Total time	Fast Intersection	0.32		Total time	Fast Intersection	0.03	0.10	0.38
	(10 Gbps)				(10 Gbps)				
	(10 00 po)				(10 00 po)				
		Fact Catar	1.20			Fact Cataor	0.22	0.96	2.05
		Fast Setup	1.20			Fast Setup	0.33	0.86	2.95
	Total time	Fast Intersection	2.48		Total time	Fast Intersection	0.24	0.47	1.38
	(100 Mbps)				(100 Mbps)				

			$ \mathbf{V} = \mathscr{Q} \cdot 2^{20}$					Y	
			$ \Lambda = \mathcal{L} \cdot \mathcal{L}$				4	16	64
		Fast Setup	0.09			Fast Setup	0.02	0.11	0.43
		Fast Intersection	0.30			Fast Intersection	0.03	0.10	0.37
	Computation				Computation				
		Fast Setup	12.63			Fast Setup	1.85	7.39	29.55
		Fast Intersection	25.25			Fast Intersection	0.66	2.65	10.59
	Communication				Communication				
One-time				Recurrent					
		Fast Setup	0.10	liceunent		Fast Setup	0.02	0.12	0.45
	Total time	Fast Intersection	0.32		Total time	Fast Intersection	0.03	0.10	0.38
	(10 Gbps)				(10 Gbps)				
	(10 00 po)				(10 00 po)				
			1.20				0.00	0.06	2.05
		Fast Setup	1.26			Fast Setup	0.33	0.86	2.95
	Total time	Fast Intersection	2.48		Total time	Fast Intersection	0.24	0.47	1.38
	(100 Mbps)				(100 Mbps)				

			$ X = \mathscr{C} \cdot 2^{20}$					Y	
			$ \Lambda = \mathcal{Z} + \mathcal{I}$				4	16	64
		Fast Setup	0.09			Fast Setup	0.02	0.11	0.43
		Fast Intersection	0.30			Fast Intersection	0.03	0.10	0.37
	Computation				Computation				
		Fast Setup	12.63			Fast Setup	1.85	7.39	29.55
	Communication	Fast Intersection	25.25			Fast Intersection	0.66	2.65	10.59
					Communication				
One-time				Recurrent					
		Fast Setup	0.10	Recuirent		Fast Setup	0.02	0.12	0.45
	Total time	Fast Intersection	0.32		Total time	Fast Intersection	0.03	0.10	0.38
	(10 Gbps)				(10 Gbns)				
	(10 00ps)				(10 00ps)				
		Fast Setup	1.26			Fast Setup	0.33	0.86	2.95
	Total time	Fast Intersection	2.48		Total time	Fast Intersection	0.24	0.47	1.38
	(100 Mbps)				(100 Mbns)				
					(100 10005)				

			$ \mathbf{v} = \boldsymbol{\mathscr{Q}} \cdot 2^2$					Y	
			$ \Lambda = \mathcal{I} \cdot 2^{-\varepsilon}$				4	16	64
		Fast Setup	0.09			Fast Setup	0.02	0.11	0.43
		Fast Intersection	0.30			Fast Intersection	0.03	0.10	0.37
	Computation	CLR [8]	1.21		Computation				
		KKRT [31]	NA						
		PSSZ [38]	NA						
		Fast Setup	12.63			Fast Setup	1.85	7.39	29.55
		Fast Intersection	25.25			Fast Intersection	0.66	2.65	10.59
	Communication	CLR [8]	0.00	Pecurrent	Communication				
		KKRT [31]	NA						
One-time		PSSZ [38]	NA						
One-time		Fast Setup	0.10	Kecuiteitt		Fast Setup	0.02	0.12	0.45
	Total time	Fast Intersection	0.32		Total time	Fast Intersection	0.03	0.10	0.38
	(10 Ghps)	CLR [8]	1.21		(10 Gbps)				
	(10 Obps)	KKRT [31]	NA		(10 Obps)				
		PSSZ [38]	NA						
		Fast Setup	1.26			Fast Setup	0.33	0.86	2.95
	Total time	Fast Intersection	2.48		Total time	Fast Intersection	0.24	0.47	1.38
	(100 Mbps)	CLR [8]	1.21		(100 Mbps)				
	(100 10098)	KKRT [31]	NA		(100 Mups)				
		PSSZ [38]	NA						

[8] Hao Chen, Kim Laine, and Peter Rindal. "Fast private set intersection from homomorphic encryption." CCS, 2017.

[31] Vladimir Kolesnikov, Ranjit Kumaresan, Mike Rosulek, and Ni Trieu. "Efficient batched oblivious PRF with applications to private set intersection." CCS, 2016. [38] Benny Pinkas, Thomas Schneider, Gil Segev, and Michael Zohner. "Phasing: Private set intersection using permutation-based hashing." USENIX Security, 2015.

		$ X = \mathscr{L} \cdot 2^{20}$							
						4	16	64	
One-time	Computation	Fast Setup	0.09	Recurrent	Computation	Fast Setup	0.02	0.11	0.43
		Fast Intersection	0.30			Fast Intersection	0.03	0.10	0.37
		CLR [8]	1.21			CLR [8]	2.21	2.21	2.21
		KKRT [31]	NA			KKRT[31]	1.62	1.62	1.62
		PSSZ [38]	NA			PSSZ [38]	0.86	0.86	0.71
	Communication	Fast Setup	12.63		Communication	Fast Setup	1.85	7.39	29.55
		Fast Intersection	25.25			Fast Intersection	0.66	2.65	10.59
		CLR [8]	0.00			CLR [8]	5.60	5.60	5.60
		KKRT [31]	NA			KKRT [31]	57.17	57.17	57.17
		PSSZ [38]	NA			PSSZ [38]	24.05	24.05	27.11
	Total time (10 Gbps)	Fast Setup	0.10		Total time (10 Gbps)	Fast Setup	0.02	0.12	0.45
		Fast Intersection	0.32			Fast Intersection	0.03	0.10	0.38
		CLR [8]	1.21			CLR [8]	2.21	2.21	2.21
		KKRT [31]	NA			KKRT [31]	1.67	1.67	1.67
		PSSZ [38]	NA			PSSZ [38]	0.88	0.88	0.73
	Total time (100 Mbps)	Fast Setup	1.26		Total time (100 Mbps)	Fast Setup	0.33	0.86	2.95
		Fast Intersection	2.48			Fast Intersection	0.24	0.47	1.38
		CLR [8]	1.21			CLR [8]	2.81	2.81	2.81
		KKRT [31]	NA			KKRT [31]	6.46	6.46	6.46
		PSSZ [38]	NA			PSSZ [38]	2.94	2.94	3.05

[8] Hao Chen, Kim Laine, and Peter Rindal. "Fast private set intersection from homomorphic encryption." CCS, 2017.

[31] Vladimir Kolesnikov, Ranjit Kumaresan, Mike Rosulek, and Ni Trieu. "Efficient batched oblivious PRF with applications to private set intersection." CCS, 2016. [38] Benny Pinkas, Thomas Schneider, Gil Segev, and Michael Zohner. "Phasing: Private set intersection using permutation-based hashing." USENIX Security, 2015.



Thank You



github.com/momalab/psi-ndss2025



Full Protocol

Protocol 2 Full Protocol

Input: S inputs set X of size |X|. \mathcal{R} inputs set Y of size |Y|. Both sets consist of bit strings of length σ_d . |X|, |Y|, and σ_d are public.

Output: S outputs \bot ; \mathcal{R} outputs $X \cap Y$.

- 1) Setup: In this step, parties select all parameters to be used in the protocol.
 - a) Security parameters: Parties agree on computational and statistical security parameters (κ and λ , respectively).
 - b) **Data compression**: From Eqs. 4 and 5 (SIV-A5), we have that $\sigma = \min(\sigma_d, \sigma_h)$. Therefore, if $\sigma_h < \sigma_d$, parties agree on a hash function $\mathcal{Z}(\cdot) : \mathbb{Z}_{2^{\sigma_h}} \mapsto \mathbb{Z}_{2^{\sigma_h}}$ for data compression. Otherwise, $\mathcal{Z}(x) = x$.
 - c) Cuckoo hashing and Permutation-based hashing:
 - i) Parties agree on the number of hash functions h and insertion depth limit d for Cuckoo hashing.
 - ii) They select the number of Cuckoo hash tables $k (\equiv \text{number of packing slots})$, and a function $g(\cdot)$ that maps a σ -bit entry x to Cuckoo hash table $T^{(g(x))}$ (SIV-B5).
 - iii) Following IV-A3 and VI-A S chooses the Cuckoo hash table size $|T^{(i)}|$ for a failure rate $\leq 2^{-\lambda}$.
 - iv) By knowing $|T^{(i)}|$, one can calculate the left $\sigma_L = \lfloor \log_2 |T^{(i)}| \rfloor$ and right $\sigma_R = \sigma \sigma_L$ sizes for Permutation-based hashing (SIV-A4).
 - v) S selects the *h* different hash functions $H : \mathbb{Z}_{2^{\sigma}} \mapsto \mathbb{Z}_{|T^{(i)}|}$, where $H_j(z_L, z_R) = z_L \oplus f_j(z_R)$ and $f_j(z_R) : \mathbb{Z}_{2^{\sigma_R}} \mapsto \mathbb{Z}_{|T^{(i)}|}$ (§IV-A4).
 - d) Encryption parameters:
 - i) Parties agree on partitioning parameters η_s and η_r (§IV-C1) and plaintext modulus $t = \prod_{i=1}^k t_i : t_i \ge 2^{\sigma_R} + 2$ (§IV-B4 and §IV-B7).
 - ii) Parties select encryption parameters $(n_s, q_s, t), (n_r, q_r, t) : n_s \ge n_r$ following Eq. 3 and V-B7 Encryption parameters are public.
 - iii) Each party generates decryption and encryption keys, which are kept private.
- 2) Cuckoo hashing: For each entry $x \in X$, S performs $z = z_L |z_R = Z(x) : |z_L| = \lfloor \log_2 |T^{(i)}| \rfloor$ and inserts z_R at $T_{H_i(z_L, z_R)}^{(g(z))} : 1 \le j \le h$. A dummy value is assigned to empty bins after inserting entries $\in X$ into tables $T^{(i)}$ (SIV-A2).
- 3) Set encryption: S encodes all Cuckoo hash tables and bins into polynomials following Eq. 8 ($\frac{1}{V-B5}$), encrypts them into ciphertexts under its own key, and sends the $\lceil \frac{|T^{(i)}|}{n} \rceil$ ciphertexts \overline{C} to \mathcal{R} .
- 4) **Computing intersection**: For each $y \in Y$, \mathcal{R} :
 - a) Computes $z = z_L | z_R = \mathcal{Z}(y) : |z_L| = \lfloor \log_2 |T^{(i)}| \rfloor$, and for each Cuckoo hash function $H_j(\cdot) \ \forall j \in \mathbb{Z} : 1 \leq j \leq h, \mathcal{R}$:
 - i) Creates a polynomial $P \in Z_t[x]/(x^{n_s}+1)$ s.t. $P_{u,v} = z_R$ for $(u,v) = (H_j(z_L, z_R) \mod n_s, g(z) 1)$ and a dummy value otherwise.
 - ii) Homomorphically computes the difference $\overline{S}^{(j)} = \overline{C} \left(\lfloor \frac{H_j(z_L, z_R)}{n_s} \rfloor \right) P.$
 - b) Performs $h \eta_s 1 : \eta_s \in \mathbb{Z}_h$ homomorphic multiplications between $\overline{S}^{(j)} : j \in \mathbb{Z}_{h+1} \setminus \{0\}$, resulting in $\eta_s + 1$ ciphertexts $\overline{M}^{(i)} \forall i \in \mathbb{Z}_{\eta_s+1}$.
 - c) Samples $\eta_s + 1$ tuples of random values $(r_1, ..., r_{n_s} \in \mathbb{Z}_t)$ from a uniform distribution, where each tuple is encoded as a polynomial $R^{(i)} \in \mathbb{Z}_t[x]/(x^{n_s} + 1)$.
 - d) Computes $\overline{D}^{(i)} = \overline{M}^{(i)} + R^{(i)} \forall i \in \mathbb{Z}_{\eta_s+1}$, modulus switches it, encrypts $R^{(i)} \mapsto \widehat{R}^{(i)}$ under its key, and sends the $(\overline{D}^{(i)}, \widehat{R}^{(i)})$ pairs to S. Note that each $\widehat{R}^{(i)}$ will be several ciphertexts if $n_s > n_r$.

- 5) **Decryption**: For all pairs $(\overline{D}^{(i)}, \widehat{R}^{(i)}) \quad \forall i \in \mathbb{Z}_{\eta_s+1}, S$:
 - a) Decrypts each $\overline{D}^{(i)} \mapsto D^{(i)}$, and homomorphically computes $\widehat{M}^{(i)} = \widehat{R}^{(i)} D^{(i)} \forall i \in \mathbb{Z}_{\eta_s+1}$.
 - b) Performs $h \eta_s 1 \eta_r : \eta_r \in \mathbb{Z}_{\eta_s+1}$ homomorphic multiplications between $\widehat{M}^{(i)} : i \in \mathbb{Z}_{\eta_s+1}$, resulting in $\eta_r + 1$ ciphertexts $\widehat{N}^{(j)} \forall j \in \mathbb{Z}_{\eta_r+1}$.
 - c) Samples $\eta_r + 1$ tuples of random values $(\rho_1, ..., \rho_{n_r} \in \mathbb{Z}_t \setminus \{\alpha \cdot t_i\} \ \forall \alpha, i \in \mathbb{Z} : 1 \le i \le k)$ such that no random value is a multiple of any packing modulus t_i , and encode each tuple as a polynomial $P^{(j)} \in \mathbb{Z}_t / (x^{n_r} + 1)$.
 - d) Homomorphically computes $\widehat{E}^{(j)} = rotate(\widehat{N}^{(j)} \cdot P^{(j)}, \omega_j) \ \forall j \in \mathbb{Z}_{\eta_r+1}$, where the function $rotate(\cdot)$, rotates the batching slots by a random number of positions $\omega_j \in \mathbb{Z}_{n_r}$, modulus switches and sends each $\widehat{E}^{(j)}$ to \mathcal{R} .
- 6) **Result**: \mathcal{R} decrypts each $\widehat{E}^{(j)} \mapsto E^{(j)}$ and outputs $X \cap Y = \bigcup_{j=1}^{|Y|} \{y_j \iff 0 \in \bigcup_{j=0}^{\eta_r} E^{(j)}\}.$

 $|X| \approx 2^{24}, |Y| = \{4, 16, 64\}$

		$ \mathbf{V} = (\ell - 2^{24})$				Y			
		$ \Lambda = \mathcal{I} \cdot 2^{-1}$				4	16	64	
One-time	Computation	Fast Setup	1.79	Recurrent	Computation	Fast Setup	0.02	0.11	0.43
		Fast Intersection	7.74			Fast Intersection	0.03	0.10	0.37
		CLR [8]	21.54			CLR [8]	23.22	23.22	23.22
		KKRT [31]	NA			KKRT[31]	30.42	40.42	30.42
		PSSZ [38]	NA			PSSZ [38]	11.11	11.11	8.54
	Communication	Fast Setup	202.04		Communication	Fast Setup	1.85	7.39	29.55
		Fast Intersection	404.08			Fast Intersection	0.66	2.65	10.59
		CLR [8]	0.00			CLR [8]	11.00	11.00	11.00
		KKRT [31]	NA			KKRT [31]	946.75	946.75	946.75
		PSSZ [38]	NA			PSSZ [38]	432.05	432.05	432.11
	Total time (10 Gbps)	Fast Setup	1.96		Total time (10 Gbps)	Fast Setup	0.02	0.12	0.45
		Fast Intersection	8.08			Fast Intersection	0.03	0.10	0.38
		CLR [8]	21.54			CLR [8]	23.22	23.22	23.22
		KKRT [31]	NA			KKRT [31]	31.21	31.21	31.21
		PSSZ [38]	NA			PSSZ [38]	11.47	11.47	8.90
	Total time (100 Mbps)	Fast Setup	18.68		Total time (100 Mbps)	Fast Setup	0.33	0.86	2.95
		Fast Intersection	41.44			Fast Intersection	0.24	0.47	1.38
		CLR [8]	21.54			CLR [8]	23.59	23.59	23.59
		KKRT [31]	NA			KKRT [31]	109.27	109.27	109.27
		PSSZ [38]	NA			PSSZ [38]	47.14	47.14	44.57

TABLE VI: Total time (in seconds) for two configurations of our PSI protocol, *Fast Setup* and *Fast Intersection*, and related work, considering a S's set size $|X| = \mathscr{L} \cdot 2^{24}$ and several sizes of \mathcal{R} 's set |Y|, where we evaluate the one-time costs and recurrent costs of performing set intersections. KKRT and PSSZ do not have one-time costs.