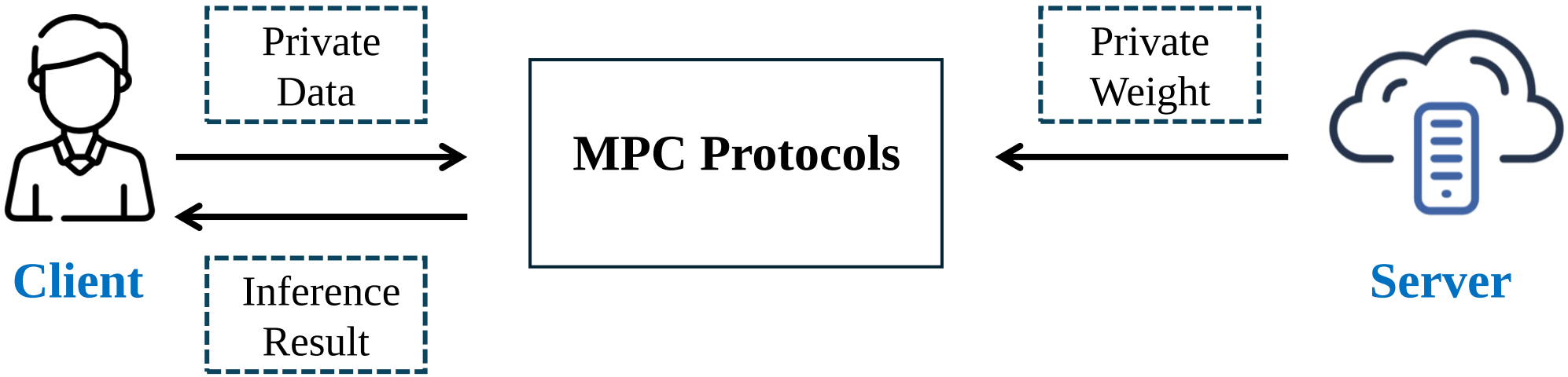
Distributed Function Secret Sharing and Applications

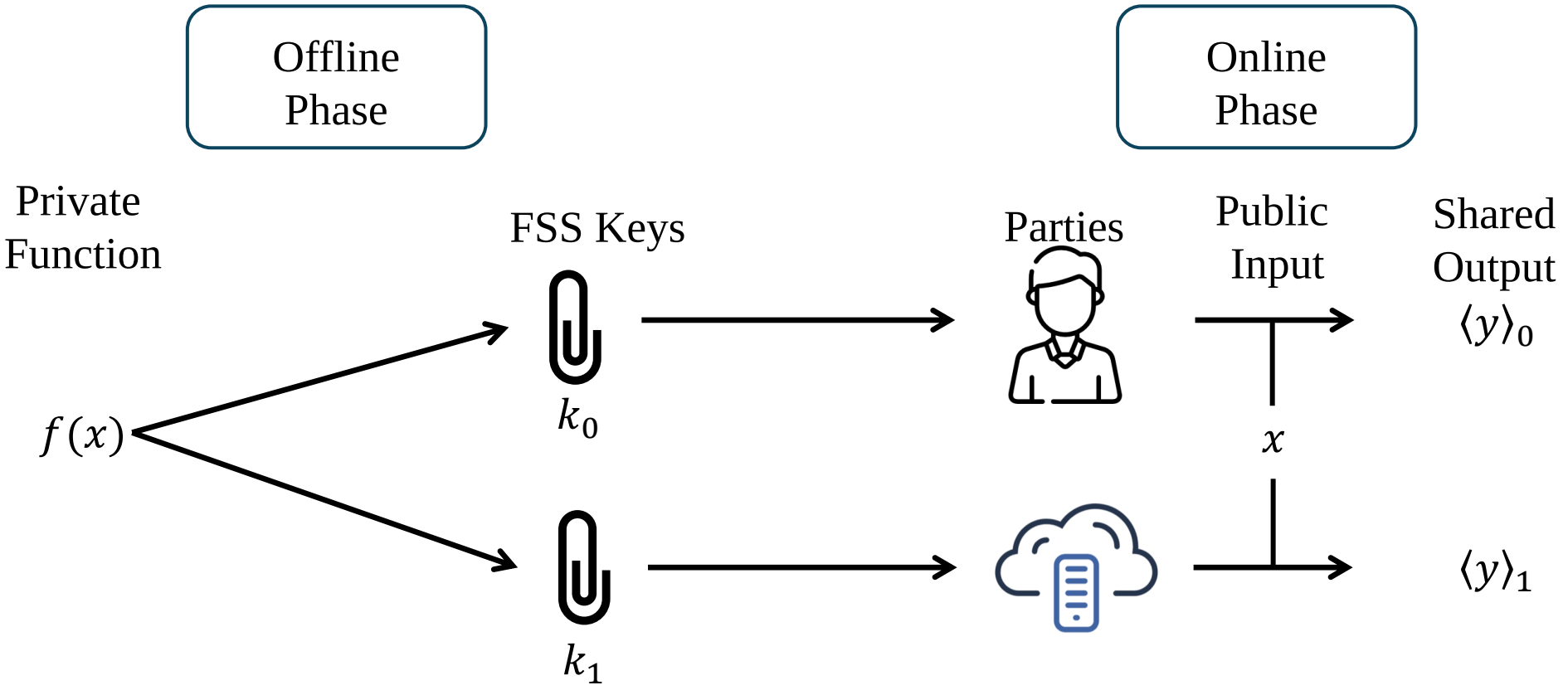
Pengzhi Xing, Hongwei Li, Meng Hao, Hanxiao Chen,
Jia Hu, Dongxiao Liu

Model Inference via Secure Multiparty Computation



- **Input:** Client secret shares the data & Server secret shares the weights.
- **Protocol Execution**: Communicate & compute in multiple rounds.
- **Output**: Client get the inference result.

Function Secret Sharing [BGI19]



- **Offline Stage**: Dealer generates FSS keys
- **Online Stage**: Parties jointly evaluate the key on the public input

Existing FSS is not Sufficient

Challenge 1:

Dealer-based FSS is unrealistic and weakens security guarantees.

- Finding a trusted party is difficult.
- A colluded dealer may compromise the privacy of honest party.

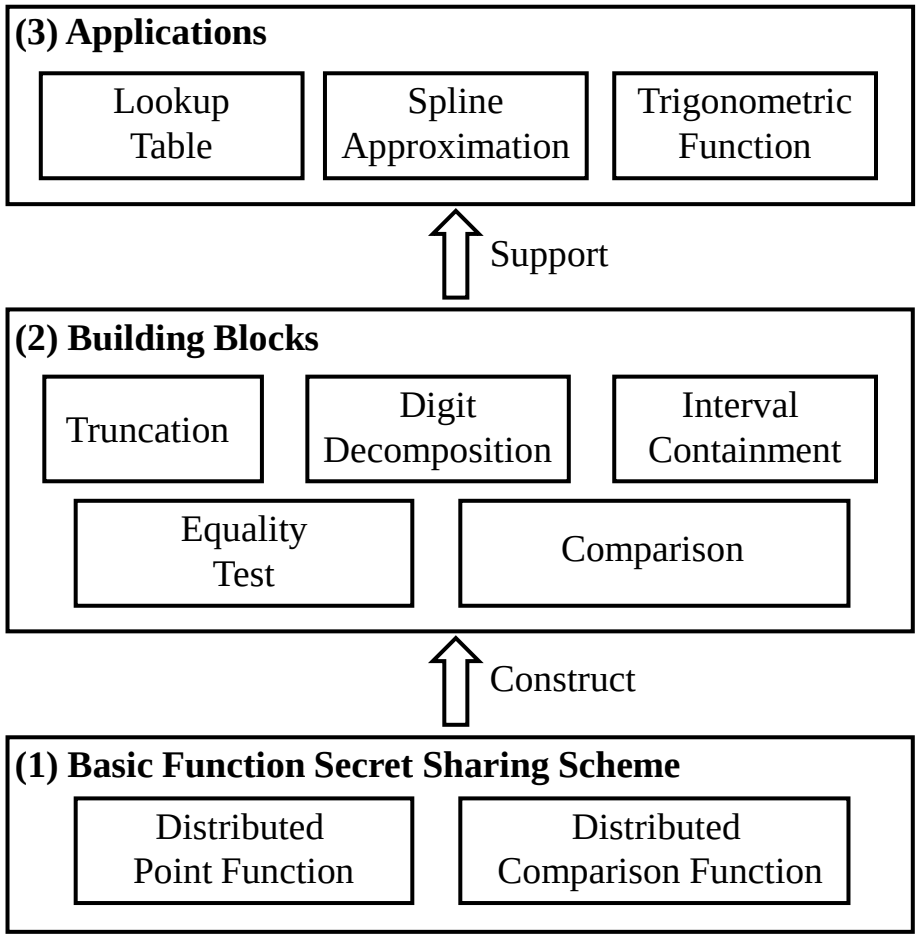
Challenge 2:

Existing dealer-less FSS have limited practicality.

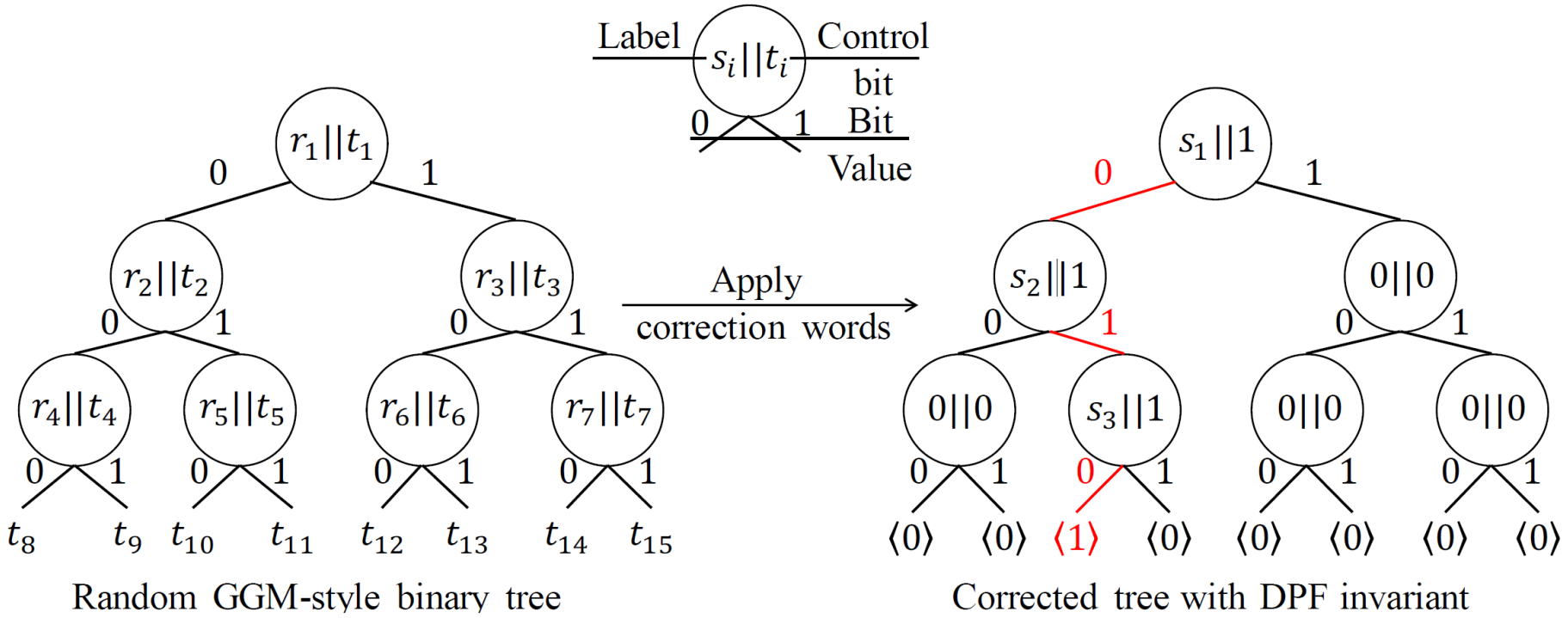
- Unable to support both arithmetic input and output.
- Potential performance bottleneck incurred by bit length.

Our Contributions

- Propose the **dealer-less FSS** scheme, including arithmetic DPF and DCF.
- FSS-based building blocks and complex function evaluation.
- Open source implementation, achieving **27~184× communication improvement** and **1.1~14× runtime improvement**



Recalling Dealer-based DPF Key Generation



Random GGM-style binary tree

Corrected tree with DPF invariant

DPF key components:

- **Root Seed**: Determine the pseudorandom GGM tree.
- **Corrections words**: Correct the tree to satisfy DPF invariant [BGI16].

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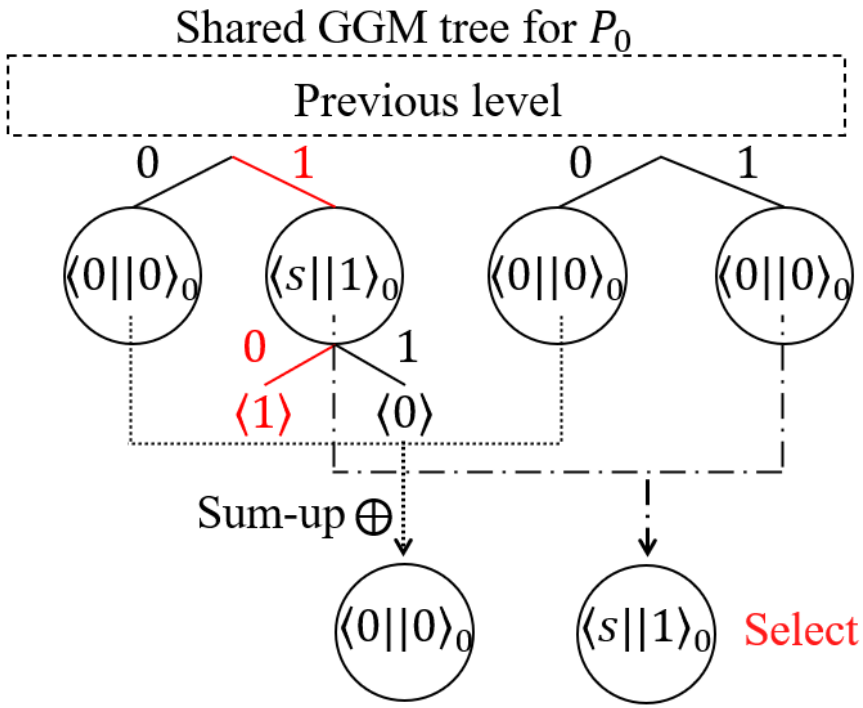
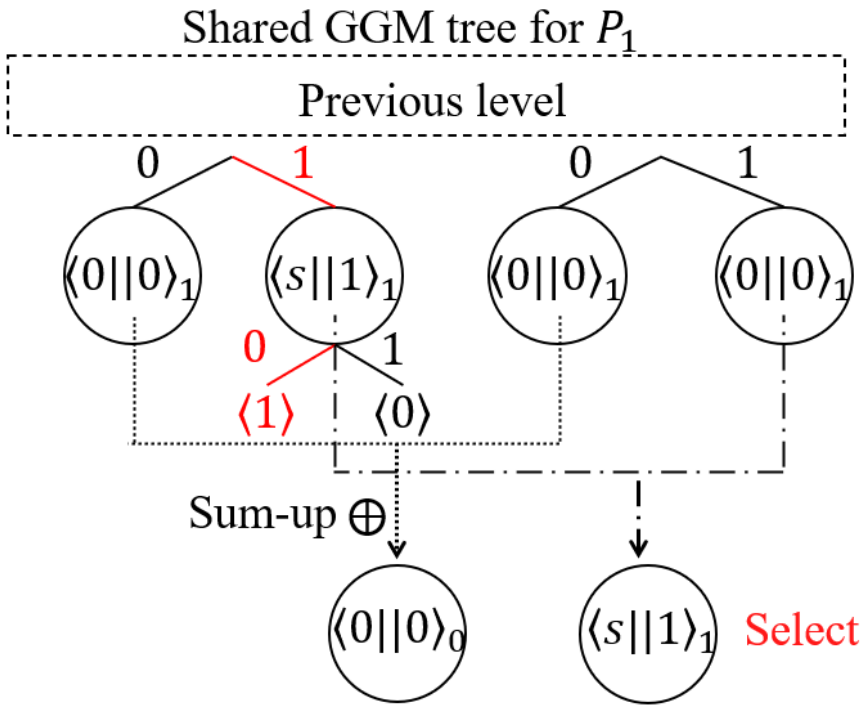
  - Solution: Sum all the left (right) nodes, off-path nodes can be cancelled.





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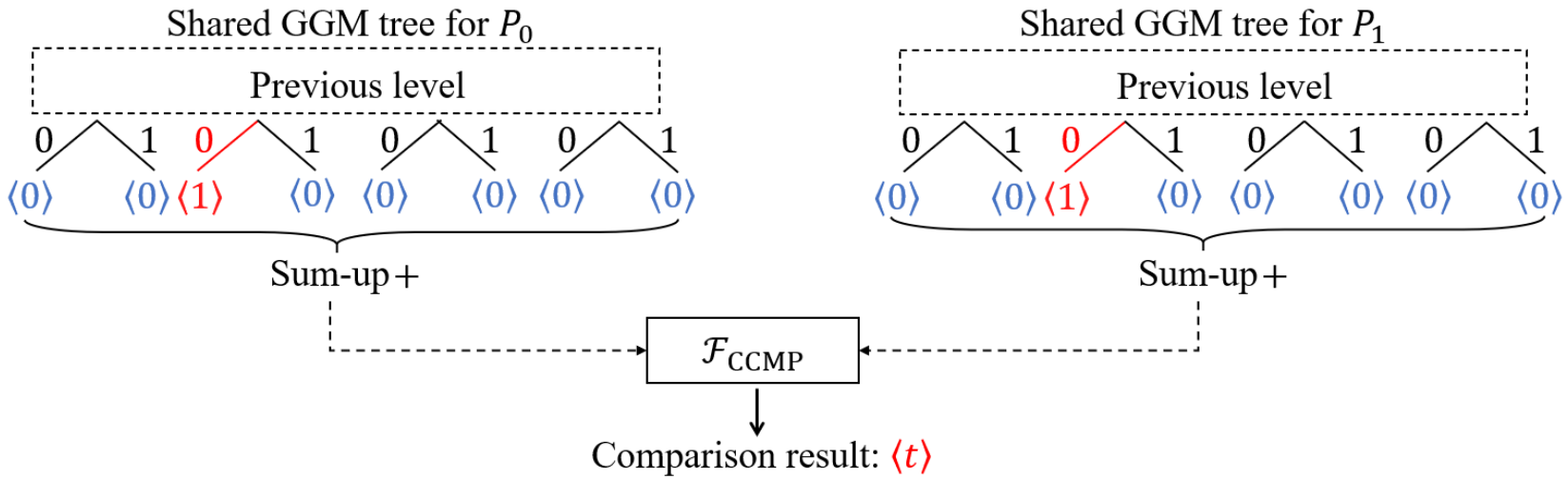
  - Solution: As $t_1 = 0$ or 1, determining which party holds the larger value suffices.



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$$1\{x_0 < x_1\} = \begin{cases} l_1 & \text{if } h_0 = h_1 \\ \neg h_1 & \text{if } h_0 \neq h_1,\ h_0 = l_0 \\ h_1 & \text{if } h_0 \neq h_1,\ h_0 \neq l_0 \end{cases}$$

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    - Just locally subtract previous multiplexer output to get what we need now.

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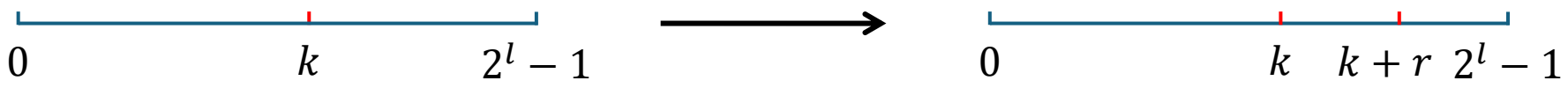

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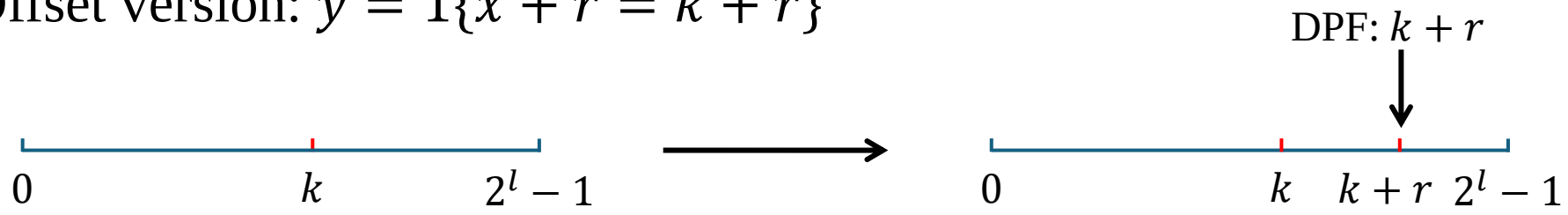

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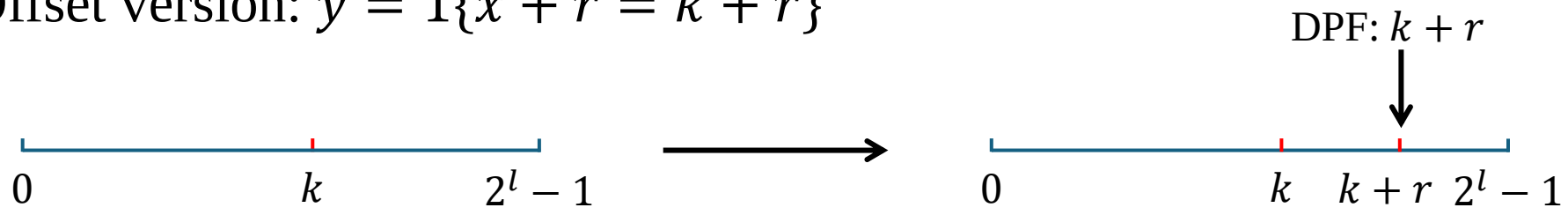

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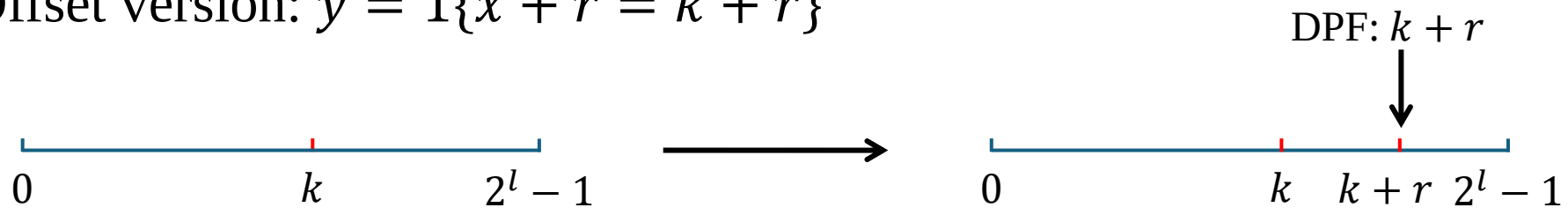

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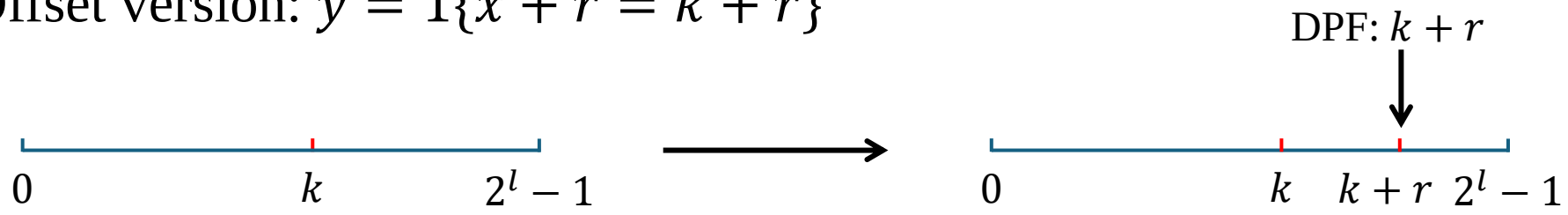

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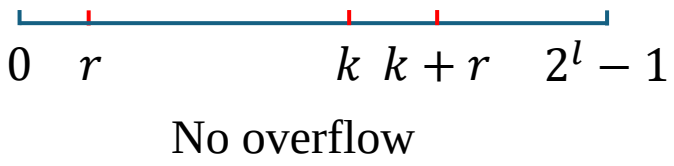
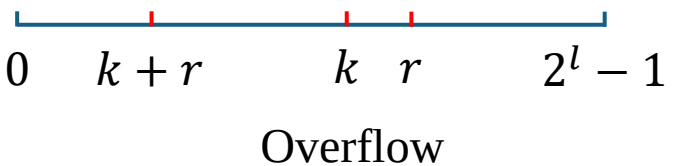


No overflow



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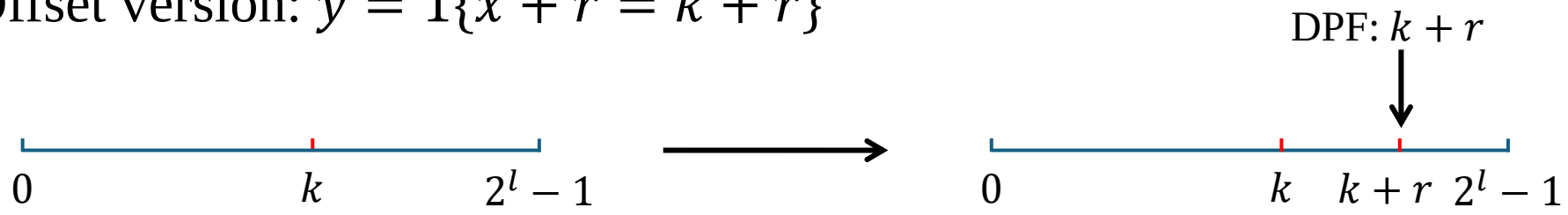

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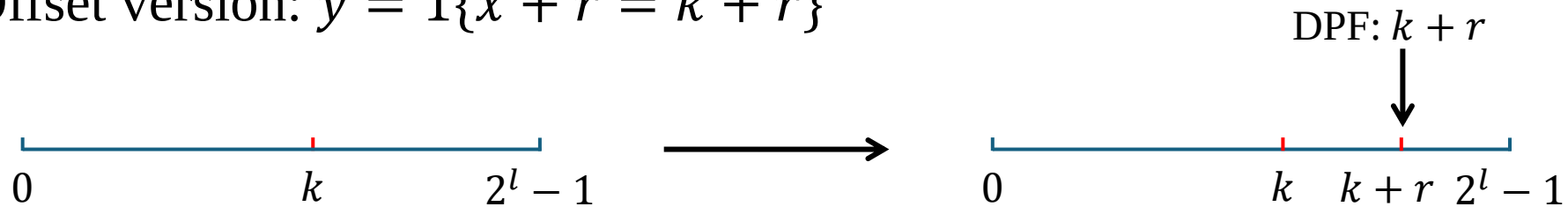

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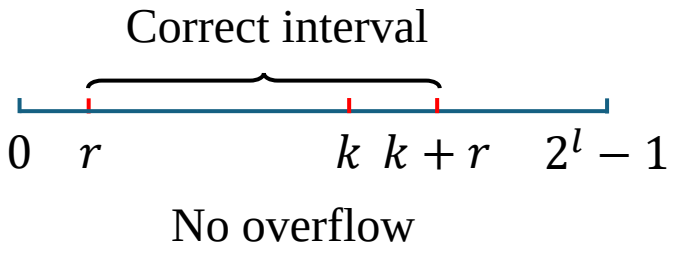

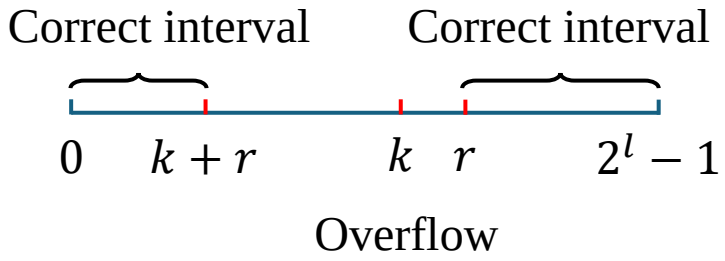

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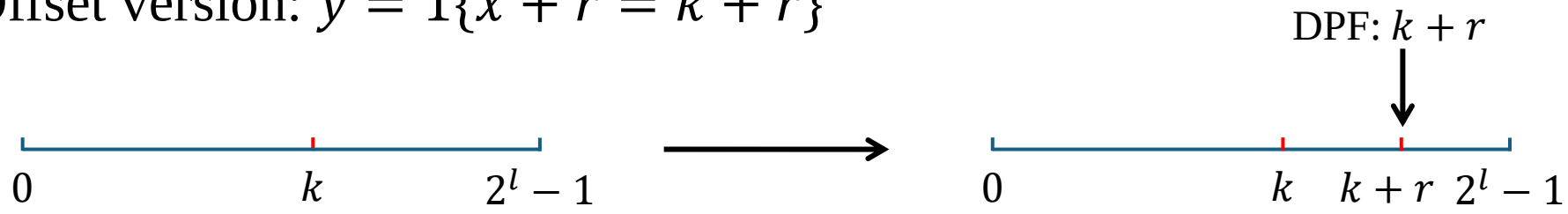

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  - Solution: Check if $k > k + r$ at offline stage.

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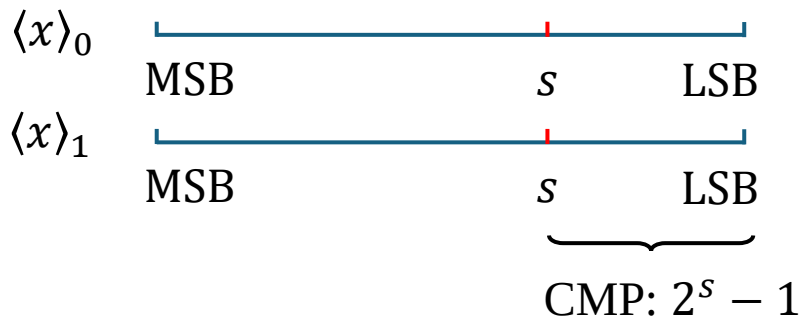

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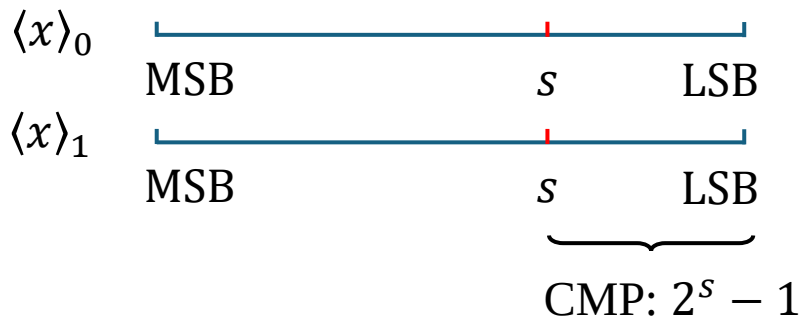

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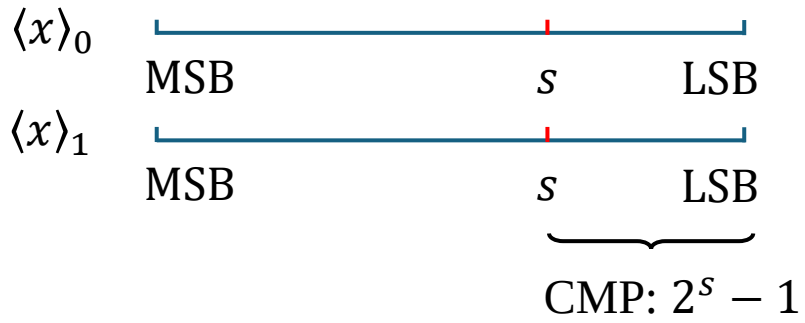

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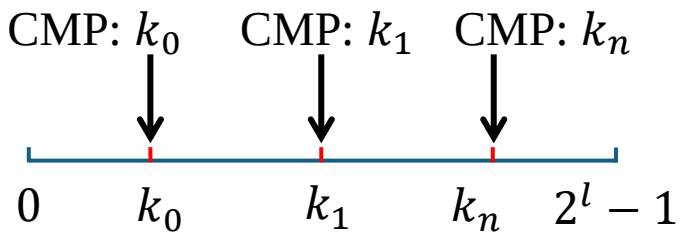

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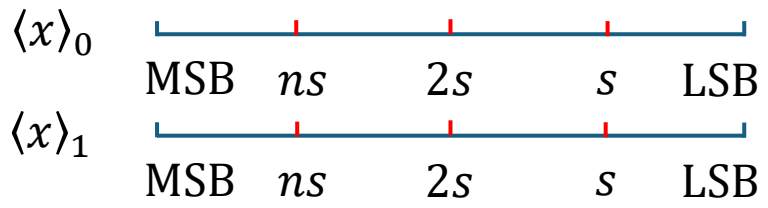

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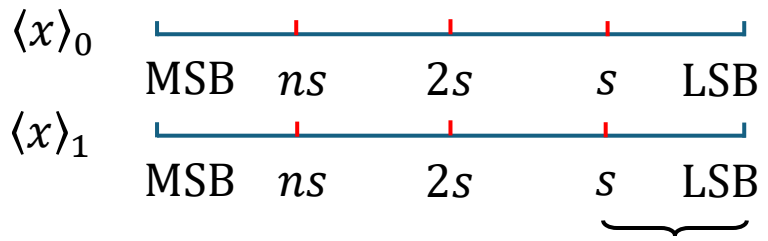

For each segment: CMP: $2^s - 1$, EQ: $2^s - 1$

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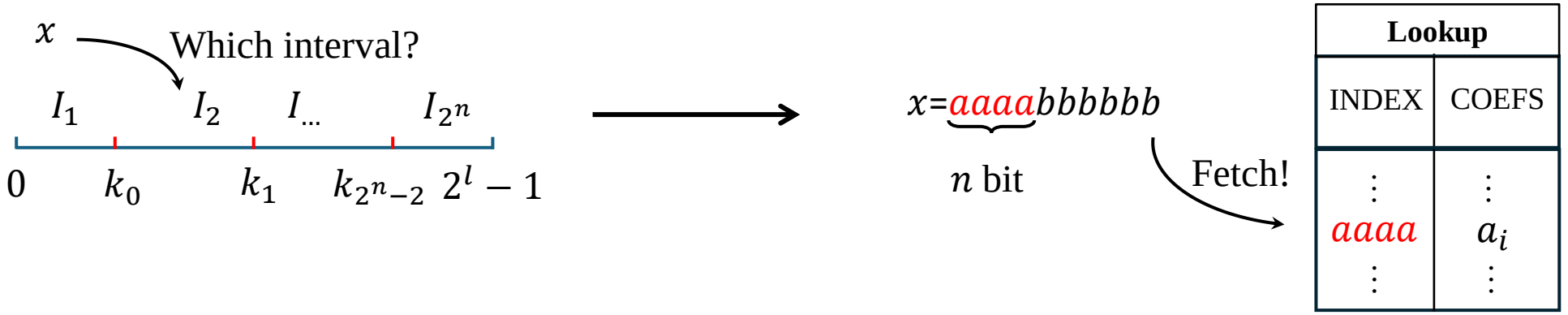

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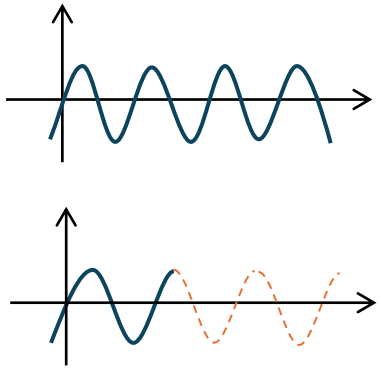

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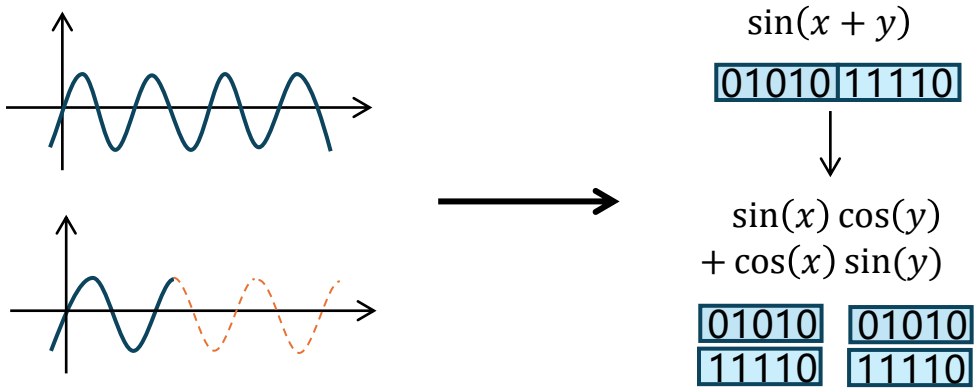

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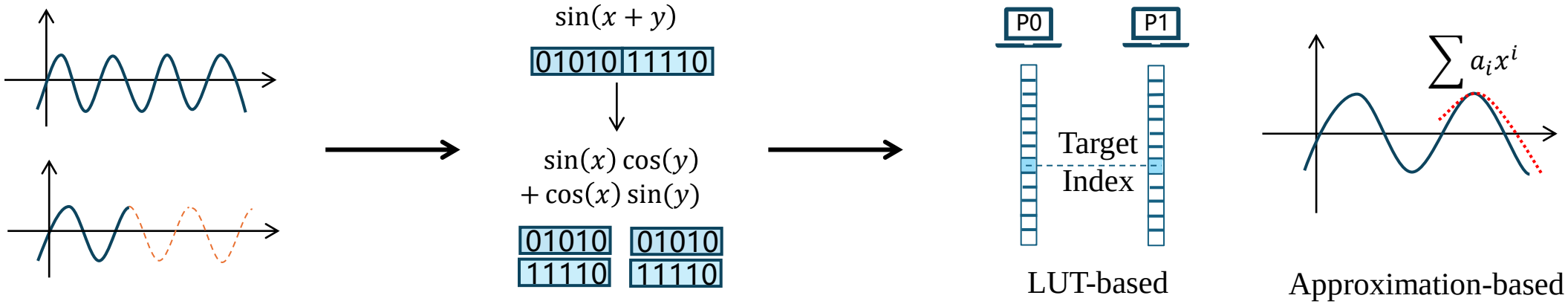

Evaluation

- **Experimental results of dealer-less DPF and DCF**

Protocol	Params.	Time LAN		Time WAN		Comm.	
	ℓ	Gen(s)	Eval(μs)	Gen(s)	Eval(μs)	Gen(MB)	Eval(B)
Π_{DPF}	8	0.050	0.577	1.288	0.615	0.046	0
	16	0.092	1.150	2.450	1.151	0.080	0
	18	0.124	1.278	2.780	1.265	0.088	0
Π_{DCF}	8	0.085	1.818	2.417	1.866	0.145	0
	16	0.180	3.357	4.625	3.505	0.277	0
	18	0.280	3.739	5.247	3.718	0.310	0

Evaluation

- **Experimental results of trigonometric evaluation**

Protocol	Impl.	Runtime		Comm.	Error
		LAN (ms)	WAN (s)	(KB)	(ULP)
Π_{Sin}	Ours (LUT)	0.612	0.164	0.050	1.477
	Ours (Approx)	**0.329**	**0.133**	**0.035**	1.387
	MP-SPDZ	0.941	0.753	1.560	0.629
	EzPC-Secfloat	1.614	0.380	26.014	**0.318**
Π_{Cos}	Ours (LUT)	0.612	0.164	0.050	0.360
	Ours (Approx)	**0.329**	**0.133**	**0.035**	1.117
	MP-SPDZ	0.892	0.713	1.528	1.070
	EzPC-Secfloat	1.625	0.382	26.080	**0.318**
Π_{Tan}	Ours (LUT)	**0.307**	**0.092**	**0.021**	**0**
	Ours (Approx)	0.309	0.133	0.025	1.053
	MP-SPDZ	1.761	1.357	3.840	5.088
	EzPC-Secfloat	2.185	0.533	36.043	0.244

Evaluation

- **Experimental results of case studies on *proximity test* and *biometric authentication***

Protocol	Impl.	Runtime		Comm.	Error
		LAN (ms)	WAN (s)	(KB)	(ULP)
Biometric Authentication	Ours (LUT)	1.429	**0.369**	**0.084**	**0**
	Ours (Approx)	**1.252**	0.530	0.100	2.549
	MP-SPDZ	5.977	5.339	15.360	3.707
	EzPC-Secfloat	9.369	2.372	165.293	2.370
Proximity Test	Ours (LUT)	2.409	0.697	0.220	2.094
	Ours (Approx)	**1.394**	**0.572**	**0.160**	**0.926**
	MP-SPDZ	1.977	1.694	9.424	2.266
	EzPC-Secfloat	8.210	2.030	173.322	6.299

Thank You

Pengzhi Xing, *p.xing@std.uestc.edu.cn*